Board Paper of Class 12-Science Term-II 2022 Math Delhi(Set 1) - Solutions

Total Time: 120

Total Marks: 40.0

Section A

Solution 1

Let, $I=\int \frac{dx}{\sqrt{4x-x^2}}$

$=\int \frac{\mathrm{d}x}{\sqrt{4x-x^2+4-4}}$

$=\int \frac{\mathrm{d}x}{\sqrt{-(x^2-4x+4)+4}}$

$=\int \frac{\mathrm{d}x}{\sqrt{2^2-(x-2)^2}}$

$=\sin^{-1}\frac{(x-2)}{2}+C \qquad \left[\because \int \frac{\mathrm{d}x}{\sqrt{a^2-(x)^2}}=\sin^{-1}\left(\frac{x}{a}\right)+C\right]$

Thus, the value of $\int \frac{dx}{\sqrt{4x-x^2}}$ is $\sin^{-1}\left(\frac{x-2}{2}\right)+C$.

Solution 2

Given that, $\frac{\mathrm{d}y}{\mathrm{d}x}=e^{x-y}+x^2e^{-y}$

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{e^x}{e^y}+\frac{x^2}{e^y}$

$\Rightarrow e^y\mathrm{d}y=\left(x^2+e^x\right)\mathrm{d}x$

On integrating both sides, we get

$\int e^y\mathrm{d}y=\int\left(x^2+e^x\right)\mathrm{d}x$

$\Rightarrow e^y=\frac{x^3}{3}+e^x+C$

Thus, the general solution of the differential equation is $e^y = \frac{x^3}{3} + e^x + C$.

Solution 3

Given that, 2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).
Let
$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4) = k$

$\Rightarrow P(X = x_1) = \frac{k}{2}$

$\text{And, } P(X = x_2) = \frac{k}{3}$

$\text{And, } P(X = x_3) = k$

$\text{And, } P(X = x_4) = \frac{k}{5}$

Since, $\sum_{i=1}^{4} P(X = x_i) = 1$
$\therefore P(X = x_1) + P(X = x_2) + P(X = x_3) + P(X = x_4) = 1$

$\Rightarrow \frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$

$\Rightarrow \frac{61}{30}k = 1$

$\Rightarrow k = \frac{30}{61}$

Thus,
$P(X = x_1) = \frac{15}{61}, P(X = x_2) = \frac{10}{61}, P(X = x_3) = \frac{30}{61} \text{ and } P(X = x_4) = \frac{6}{61}$.

Solution 4

Given: $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k},\ \overrightarrow{a} \cdot \overrightarrow{b} = 1,\ \overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}$

$\text{Let } \overrightarrow{b} = x\hat{i} + y\hat{j} + z\hat{k}$
$\because \overrightarrow{a} \cdot \overrightarrow{b} = 1$

$\Rightarrow \left(\hat{i} + \hat{j} + \hat{k}\right) \cdot \left(x\hat{i} + y\hat{j} + z\hat{k}\right) = 1$

$\Rightarrow x + y + z = 1 \qquad \ldots\ldots(1)$

$\because \vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow \hat{i}\,(z-y) - \hat{j}\,(z-x) + \hat{k}\,(y-x) = \hat{j} - \hat{k}$$

On comparing the coefficients in both the sides, we get
$(z-y)=0,\ -1\,(z-x)=1,\ (y-x)=-1$

$\Rightarrow z=y,\ z=x-1,\ y=-1+x \quad \ldots\ldots(2)$

From (1) and (2) we get
$x + x - 1 + x - 1 = 1$
$\Rightarrow x = 1$
$\Rightarrow y = z = 0$
$\therefore \vec{b} = 1\hat{i} + 0\hat{j} + 0\hat{k}$

$$\Rightarrow \left|\vec{b}\right| = \sqrt{1^2+0^2+0^2}$$

$$\Rightarrow \left|\vec{b}\right| = 1$$

Solution 5

Since the line makes an angle of α, β and γ with the coordinate axis,
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$

$$\begin{aligned}\therefore \cos(2\alpha) + \cos(2\beta) + \cos(2\gamma) &= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 \\ &= 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma - 3 \\ &= 2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right) - 3 \\ &= 2 - 3 \\ &= -1\end{aligned}$$

Hence, the value of $\cos(2\alpha) + \cos(2\beta) + \cos(2\gamma)$ is -1.

Solution 6

$$\mathrm{P}\left(\overline{\mathrm{A}} \cup \overline{\mathrm{B}}\right)=\mathrm{P}\left(\overline{\mathrm{A} \cap \mathrm{B}}\right)$$
$$\Rightarrow \frac{1}{4}=1-\mathrm{P}(\mathrm{A} \cap \mathrm{B})$$
$$\Rightarrow \mathrm{P}(\mathrm{A} \cap \mathrm{B})=1-\frac{1}{4}$$
$$\Rightarrow \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\frac{3}{4}$$

Two events A and B are independent if $\mathrm{P}(\mathrm{A} \cap \mathrm{B})=\mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B})$.

$$\text{Now, } \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B})=\frac{1}{2} \times \frac{7}{12}$$
$$=\frac{7}{24}$$

Since $\mathrm{P}(\mathrm{A} \cap \mathrm{B}) \neq \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B})$, the given events are not independent.

OR

Box B_1 contains 1 white ball, 3 red balls and box B_2 contains 2 white balls, 3 red balls.
A ball is drawn from each of the boxes thus, the probability that both drawn balls have same colour is equal to the sum of probability that both are white and the probability that both are red.

$$\therefore \text{Required probability}=\mathrm{P}(\text{both white})+\mathrm{P}(\text{both red})$$
$$=\frac{1}{4} \times \frac{2}{5}+\frac{3}{4} \times \frac{3}{5}$$
$$=\frac{2}{20}+\frac{9}{20}$$
$$=\frac{11}{20}$$

Hence, the probability that both the drawn balls are of same colour is $\frac{11}{20}$.

Section B

Solution 7

Let $I=\int_0^{\frac{\pi}{4}} \frac{1}{1+\tan x} \mathrm{~d} x$.

$$I=\int_0^{\frac{\pi}{4}} \frac{1}{1+\tan x}\,\mathrm{d}\,x$$

$$=\int_0^{\frac{\pi}{4}} \frac{1}{1+\tan\left(\frac{\pi}{4}-x\right)}\,\mathrm{d}\,x \qquad \left[\text{Using } \int_0^a f(x)\,\mathrm{d}\,x = \int_0^a f(a-x)\,\mathrm{d}\,x\right]$$

$$=\int_0^{\frac{\pi}{4}} \frac{1}{1+\frac{1-\tan x}{1+\tan x}}\,\mathrm{d}\,x$$

$$=\int_0^{\frac{\pi}{4}} \frac{1+\tan x}{2}\,\mathrm{d}\,x$$

$$=\int_0^{\frac{\pi}{4}} \frac{1}{2}\,\mathrm{d}\,x + \int_0^{\frac{\pi}{4}} \frac{\tan x}{2}\,\mathrm{d}\,x$$

$$=\left[\frac{x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{1}{2}\ln(\sec x)\right]_0^{\frac{\pi}{4}}$$

$$=\frac{\frac{\pi}{4}-0}{2} + \frac{1}{2}\left[\ln\left(\sec\frac{\pi}{4}\right) - \ln(\sec 0)\right]$$

$$=\frac{\pi}{8} + \frac{1}{2}\left[\ln\left(\sqrt{2}\right) - \ln(1)\right]$$

$$=\frac{\pi}{8} + \frac{1}{2}\ln\left(\sqrt{2}\right)$$

$$=\frac{\pi}{8} + \frac{1}{4}\ln(2)$$

Solution 8

Let $\overrightarrow{a} = a_1\,\hat{i} + a_2\,\hat{j} + a_3\,\hat{k}$ and $\overrightarrow{b} = b_1\,\hat{i} + b_2\,\hat{j} + b_3\,\hat{k}$

Then, $\left|\overrightarrow{a}\right| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$ and $\left|\overrightarrow{b}\right| = \sqrt{(b_1)^2 + (b_2)^2 + (b_3)^2}$

Now, $\overrightarrow{a} + \overrightarrow{b} = (a_1 + b_1)\,\hat{i} + (a_2 + b_2)\,\hat{j} + (a_3 + b_3)\,\hat{k}$

$$\therefore \left|\overrightarrow{a} + \overrightarrow{b}\right| = \sqrt{(a_1+b_1)^2 + (a_2+b_2)^2 + (a_3+b_3)^2}$$

$$= \sqrt{(a_1)^2 + (b_1)^2 + 2a_1b_1 + a_2{}^2 + b_2{}^2 + 2a_2b_2 + a_3{}^2 + b_3{}^2 + 2a_3b_3}$$

Now, given that

$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|$, therefore,

$$\sqrt{(a_1)^2 + (b_1)^2 + 2a_1b_1 + a_2{}^2 + b_2{}^2 + 2a_2b_2 + a_3{}^2 + b_3{}^2 + 2a_3b_3} = \sqrt{(b_1)^2 + (b_2)^2$$

$$\Rightarrow (a_1)^2 + (b_1)^2 + 2a_1b_1 + a_2{}^2 + b_2{}^2 + 2a_2b_2 + a_3{}^2 + b_3{}^2 + 2a_3b_3 = (b_1)^2 + (b_2)^2 +$$

$$\Rightarrow (a_1)^2 + 2a_1b_1 + a_2{}^2 + 2a_2b_2 + a_3{}^2 + 2a_3b_3 = 0 \quad .....(1)$$

Consider

$$\left(\vec{a} + 2\vec{b}\right).\ \vec{a}$$

$$= \left(a_1\ \hat{i} + a_2\ \hat{j} + a_3\ \hat{k}\ + 2b_1\ \hat{i} + 2b_2\ \hat{j} + 2b_3\ \hat{k}\right).\left(a_1\ \hat{i} + a_2\ \hat{j} + a_3\ \hat{k}\right) = 0$$

$$= a_1{}^2 + 2a_1b_1 + a_2{}^2 + 2a_2b_2 + a_3{}^2 + 2a_3b_3$$

$$= 0 \qquad [\text{From } (1)]$$

Thus, $\left(\vec{a} + 2\vec{b}\right)$ is perpendicular to $\vec{a}$.

OR

Since $\vec{a}$ and $\vec{b}$ are unit vectors, $\left|\vec{a}\right| = \left|\vec{b}\right| = 1.$

$$\therefore \left|\vec{a} - \vec{b}\right| = \sqrt{\left(\vec{a} - \vec{b}\right)^2}$$

$$= \sqrt{\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta}$$

$$= \sqrt{1^2 + 1^2 - 2\cos(\theta)}$$

$$= \sqrt{2 - 2\cos\theta}$$

$$= \sqrt{2(1 - \cos\theta)}$$

$$= \sqrt{2 \times 2\sin^2\frac{\theta}{2}}$$

$$= 2\sin\frac{\theta}{2}$$

$$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$$

Hence proved.

Solution 9

The vector equation of the plane passing through the intersections of planes

$\overrightarrow{r} \cdot \overrightarrow{n}_1 = d_1$ and $\overrightarrow{r} \cdot \overrightarrow{n}_2 = d_2$ is of the form $\overrightarrow{r} \cdot \left(\overrightarrow{n}_1 + \lambda \overrightarrow{n}_2\right) = d_1 + \lambda d_2$.

Here, $\overrightarrow{n}_1 = \left(\hat{i} + \hat{j} + \hat{k}\right)$, $\overrightarrow{n}_2 = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$, $d_1 = 10 \text{ and } d_2 = -4$.

Therefore, the required equation is

$$\overrightarrow{r} \cdot \left[\left(\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - \hat{k}\right)\right] = 10 - 4\lambda$$

$$\overrightarrow{r} \cdot \left[(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}\right] = 10 - 4\lambda \qquad .....(1)$$

Now, substitute $\overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}$,

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left[(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k}\right] = 10 - 4\lambda$$

$$\Rightarrow x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) = 10 - 4\lambda \quad .....(2)$$

The plane (2) passes through the point $(x_1,\ y_1,\ z_1) = (-2,\ 3,\ 1)$.

$$\Rightarrow (-2)(1 + 2\lambda) + 3(1 + 3\lambda) + 1(1 - \lambda) = 10 - 4\lambda$$

$$\Rightarrow -2 - 4\lambda + 3 + 9\lambda + 1 - \lambda = 10 - 4\lambda$$

$$\Rightarrow 2 + 4\lambda = 10 - 4\lambda$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1 \qquad .....(3)$$

Substitute (3) in (1), we get

$$\overrightarrow{r} \cdot \left[(1 + 2 \times 1)\hat{i} + (1 + 3 \times 1)\hat{j} + (1 - 1)\hat{k}\right] = 10 - 4 \times 1$$

$$\Rightarrow \overrightarrow{r} \cdot \left[3\hat{i} + 4\hat{j}\right] = 10 - 4$$

$$\Rightarrow \overrightarrow{r} \cdot \left(3\hat{i} + 4\hat{j}\right) = 6$$

Hence, the required equation of the plane is $\overrightarrow{r} \cdot \left(3\hat{i} + 4\hat{j}\right) = 6$.

Solution 10

$$\int uv dx = u \int v dx - \int \left\{\frac{du}{dx} \int v dx\right\} dx$$

Let $v = \sin 2x$ and $u = e^x$

Then,

$$\int e^x \sin 2x = e^x \left(\int \sin 2x dx\right) - \int \left\{\frac{d(e^x)}{dx} \int \sin 2x dx\right\} dx \qquad \left[\int uvdx = u \int vdx - \int\right.$$

$$= \frac{-1}{2} e^x \cos(2x) - \int \frac{-1}{2} e^x \cos(2x) dx$$

$$= \frac{-1}{2} e^x \cos(2x) + \frac{1}{2} \int e^x \cos(2x) dx$$

$$= \frac{-1}{2} e^x \cos(2x) + \frac{1}{2} \left(\frac{e^x \sin(2x)}{2} - \frac{1}{2} \int e^x \sin(2x) dx\right)$$

$$= \frac{-1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x) - \frac{1}{4} \int e^x \sin(2x) dx$$

$$\Rightarrow \int e^x \sin 2x dx + \frac{1}{4} \int e^x \sin 2x dx = \frac{-1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x)$$

$$\Rightarrow \frac{5}{4} \int e^x \sin 2x dx = \frac{-1}{2} e^x \cos(2x) + \frac{1}{4} e^x \sin(2x)$$

$$\Rightarrow \int e^x \sin 2x dx = \frac{-2}{5} e^x \cos(2x) + \frac{1}{5} e^x \sin(2x) + C$$

$$\Rightarrow \int e^x \sin 2x dx = \frac{e^x}{5} [\sin(2x) - 2\cos(2x)] + C$$

OR

Let $x^2 + 1 = t$.
$\Rightarrow 2xdx = dt$
So,

$$\int \frac{2x}{(x^2+1)(x^2+2)} dx = \int \frac{dt}{t(t+1)}$$

$$= \int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \int \frac{dt}{t} - \int \frac{dt}{t+1}$$

$$= \log|t| - \log|t+1| + C$$

$$= \log\left|\frac{t}{t+1}\right| + C$$

$$= \log\left(\frac{x^2+1}{x^2+2}\right) + C$$

Section C

Solution 11

Given that, the chances of selection of A, B and C are in the ratio 1 : 2 : 4.
Let the events be defined as:
Q: No change takes place
E_1: Person A gets appointed
E_2: Person B gets appointed
E_3: Person C gets appointed

Now,

$\mathrm{P}\left(\mathrm{E}_1\right)=\frac{1}{7}$

$\mathrm{P}\left(\mathrm{E}_2\right)=\frac{2}{7}$

$\mathrm{P}\left(\mathrm{E}_3\right)=\frac{4}{7}$

It is given that the probabilities of A, B and C introducing changes to improve profits of company are 0.8, 0.5 and 0.3 respectively.
Thus, the probabilities of no changes in the profits on appointment of A, B and C are 0.2, 0.5 and 0.7 respectively.

Therefore,

$\mathrm{P}\left(\mathrm{Q}|\mathrm{E}_1\right)=0.2=\frac{2}{10}$

$\mathrm{P}\left(\mathrm{Q}|\mathrm{E}_2\right)=0.5=\frac{5}{10}$

$\mathrm{P}\left(\mathrm{Q}|\mathrm{E}_3\right)=0.7=\frac{7}{10}$

Using Bayes' theorem,

$$\mathrm{P}\left(\mathrm{E}_1|\mathrm{Q}\right)=\frac{\mathrm{P}(\mathrm{Q}|\mathrm{E}_1)\cdot\mathrm{P}(\mathrm{E}_1)}{\mathrm{P}(\mathrm{Q}|\mathrm{E}_1)\cdot\mathrm{P}(\mathrm{E}_1)+\mathrm{P}(\mathrm{Q}|\mathrm{E}_2)\cdot\mathrm{P}(\mathrm{E}_2)+\mathrm{P}(\mathrm{Q}|\mathrm{E}_3)\cdot\mathrm{P}(\mathrm{E}_3)}$$

$$=\frac{\frac{2}{10}\times\frac{1}{7}}{\frac{2}{10}\times\frac{1}{7}+\frac{5}{10}\times\frac{2}{7}+\frac{7}{10}\times\frac{4}{7}}$$

$$=\frac{2}{2+10+28}$$

$$=\frac{2}{40}$$

$$=\frac{1}{20}$$

Hence, if increase in the profit does not take place, then the probability that it is due to the appointment of A is $\frac{1}{20}$.

Solution 12

Given that, the area is bounded by y = 1 and y = $|x-1|$.

At y = 1,

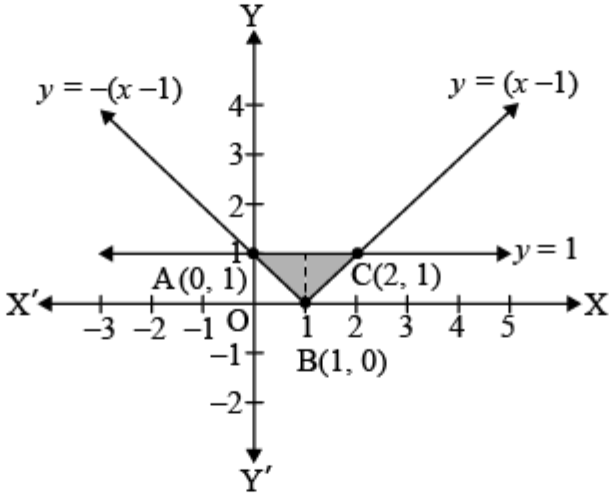
$y=x-1$

$\Rightarrow 1=x-1$

$\Rightarrow x=2$

The points of intersection of the two curves is given as:
A(0, 1), B(1, 0) and C(2, 1)

For the two curves, the bounded area is as follows:



Now,

$$\text{Area bounded by the two curves} = \left|\int_a^b y\,dx\right|$$

$$= \left|\int_0^2 1\,dx\right| - \left|\int_0^1 [-(x-1)]\,dx\right| - \left|\int_1^2 (x-1$$

$$= \left|\int_0^2 1\,dx\right| - \left|\int_0^1 (x-1)\,dx\right| - \left|\int_1^2 (x-1)\,d$$

$$= \left|\,x\Big|_0^2\right| - \left|\frac{x^2}{2} - x\Big|_0^1\right| - \left|\frac{x^2}{2} - x\Big|_1^2\right|$$

$$= |2-0| - \left|\frac{1}{2} - 1\right| - \left|2 - 2 - \frac{1}{2} + 1\right|$$

$$= 2 - \frac{1}{2} - \frac{1}{2}$$

$$= 1$$

Hence, the area bounded by the two curves is 1 sq units.

Solution 13

$$\left(y - \sin^2 x\right)dx + \tan x dy = 0$$

$$\Rightarrow y - \sin^2 x + \tan x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{y}{\tan x} - \frac{\sin^2 x}{\tan x} + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} + y \cot x = \sin x \cos x$$

Thus, the differential equation is Linear differential equation.

Compare the equation with the standard form $\frac{dy}{dx} + Py = Q$, where *P* and *Q* are the functions of *x* only.

So, P = cot$x$ and Q = sin$x$ cos$x$

Integrating factor (IF) = $e^{\int Pdx}$

$$\therefore \text{IF} = e^{\int \cot x dx}$$
$$= e^{\log \ \sin x}$$
$$= \sin x \qquad \left[\because e^{\log x} = x\right]$$

Thus, the solution of the differential equation is given by

$$\therefore \text{IF} \times y = \int Q \times \text{IF} \ \ \mathrm{d}\,x$$
$$\Rightarrow y \sin x = \int \sin x \cos x \times \sin x \,\mathrm{d}\,x$$
$$\Rightarrow y \sin x = \int \sin^2 x \cos x \,\mathrm{d}\,x$$

Put sin$x$ = t, so cos$x$ dx = dt

$$y \sin x = \int t^2 dt$$
$$\Rightarrow y \sin x = \frac{t^3}{3} + C$$
$$\Rightarrow y \sin x = \frac{\sin^3 x}{3} + C$$
$$\Rightarrow y = \frac{\sin^2 x}{3} + C \ \cos ecx$$

Hence, the solution of the differential equation is $y = \frac{\sin^2 x}{3} + C \ \cos ecx$.

OR

$$\left(x^3 + y^3\right) dy = x^2 y dx$$
$$\Rightarrow \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = \frac{x^2 y}{x^3 + y^3} \quad .....(1)$$

Since the numerator and denominator are functions of degree 3, so the differential equation is homogeneous.

Put y = vx.

$$\Rightarrow \frac{\mathrm{d}\,y}{\mathrm{d}\,x} = v + x\frac{\mathrm{d}\,v}{\mathrm{d}\,x} \qquad .....(2)$$

Substitute (2) in (1)

$v+x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{vx^3}{x^3+v^3x^3}$

$\Rightarrow v+x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{v}{1+v^3}$

$\Rightarrow x\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{v}{1+v^3}-v$

$\Rightarrow x\frac{\mathrm{d}v}{\mathrm{d}x}=-\frac{v^4}{1+v^3}$

$\Rightarrow \frac{1+v^3}{v^4}dv=-\frac{dx}{x}$

$\Rightarrow \left(\frac{1}{v^4}+\frac{1}{v}\right)dv=-\frac{dx}{x}$

Integrating both the sides, we get

$\Rightarrow \frac{v^{-3}}{-3}+\log|v|=-\log|x|+C$

$\Rightarrow -\frac{1}{3v^3}+\log|v|=-\log|x|+C$

$\Rightarrow -\frac{1}{3}\times\frac{x^3}{y^3}+\log\left|\frac{y}{x}\right|+\log|x|=C$

$\Rightarrow -\frac{x^3}{3y^3}+\log|y|=C$

Which is the general solution of the given differential equation.

Solution 14

(a) Given:

$\vec{r}=\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$

and $\vec{r}=3\hat{i}+3\hat{j}+\mu\left(2\hat{i}+\hat{j}+\hat{k}\right)$

We know that the shortest distance between two lines

$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$ is given by

$$d=\left|\frac{\left(\vec{b_1}\times\vec{b_2}\right)\cdot\left(\vec{a_2}-\vec{a_1}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

Here,

$\vec{a_1}=0\hat{i}+0\hat{j}+0\hat{k},\ \vec{b_1}=\hat{i}+2\hat{j}-\hat{k},\ \vec{a_2}=3\hat{i}+3\hat{j}\ and\ \vec{b_2}=2\hat{i}+\hat{j}+\hat{k}$

$\vec{a_2}-\vec{a_1}=3\hat{i}+3\hat{j}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= (2+1)\hat{i} - (1+2)\hat{j} + (1-4)\hat{k}$$

$$= 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) \cdot \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(3\hat{i} - 3\hat{j} - 3\hat{k}\right) \cdot \left(3\hat{i} + 3\hat{j}\right)$$

$$= 9 - 9$$

$$= 0$$

$$\therefore \left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Here, $\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) \cdot \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = 0$

$$\therefore d = \left| \frac{\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) \cdot \left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|} \right|$$

$$\Rightarrow d = 0$$

Hence, the shortest distance between both lines is 0 units.

(b) The cartesian equation of the line of motorcycle A is

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = p\left(\text{Let}\right)$$

$$\Rightarrow (x, y, z) = (p, 2p, -p)$$

The cartesian equation of the line of motorcycle B is

$$\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1} = q\left(\text{Let}\right)$$

$$\Rightarrow (x, y, z) = (2q+3, q+3, q)$$

For the collision of the motorcycles points $(p, 2p, -p)$ and $(2q + 3, q + 3, q)$ should be equal.

$$\therefore p = 2q + 3 \qquad \ldots\ldots(1)$$
$$2p = q + 3$$
$$\text{and } \; -p = q \qquad \ldots\ldots(2)$$
Solving (1) and (2), we get

$q = -1$ and $p = 1$

Hence, the point of collision will be (1, 2, −1).